C H A P T E R 8

Radiation Heat Transfer

8-1 INTRODUCTION

Preceding chapters have shown how conduction and convection heat transfer may be calculated with the aid of both mathematical analysis and empirical data. We now wish to consider the third mode of heat transfer—thermal radiation.

Thermal radiation **is that electromagnetic radiation emitted by a body as a result of its temperature.**

8-2 PHYSICAL MECHANISM

we say that it is propagated at the speed of light, 3×10^8 m/s. This

speed is equal to the product of the wavelength and frequency of the radiation,

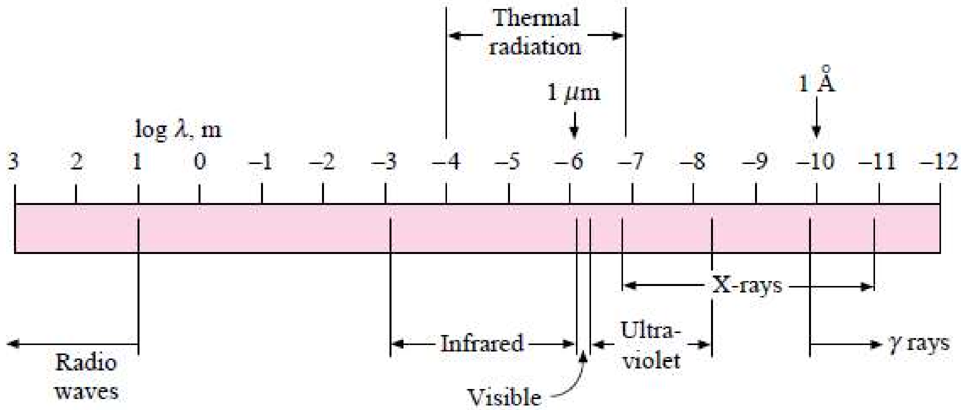
$$c = \lambda \nu$$

where

c = speed of light
λ = wavelength
ν = frequency

The unit for λ may be centimeters, angstroms ($1\ \text{Å} = 10^{-8}$ cm), or micrometers ($1\ \mu\text{m} = 10^{-6}$ m). A portion of the electromagnetic spectrum is shown in Figure 8-1. Thermal radiation lies in the range from about 0.1 to 100 μm, while the visible-light portion of the spectrum is very narrow, extending from about 0.35 to 0.75 μm.

Figure 8-1 | Electromagnetic spectrum.



The propagation of thermal radiation takes place in the form of discrete quanta, each quantum having an energy of

$$E = h\nu \quad [8\text{-}1]$$

where h is Planck's constant and has the value

$$h = 6.625 \times 10^{-34}\ \text{J}\cdot\text{s}$$

is integrated over all wavelengths, the total energy emitted is proportional to absolute temperature to the fourth power:

$$E_b = \sigma T^4 \quad [8\text{-}3]$$

Equation (8-3) is called the Stefan-Boltzmann law, E_b is the energy radiated per unit time and per unit area by the ideal radiator, and σ is the Stefan-Boltzmann constant, which has the value

$$\sigma = 5.669 \times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4 \quad [0.1714 \times 10^{-8}\ \text{Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{R}^4]$$

where E_b is in watts per square meter and T is in degrees Kelvin. In the thermodynamic

Radiation Shape Factor

F_{1-2} : fraction of the energy leaving surface 1 which reaches surface 2.

F_{2-1} : fraction of the energy leaving surface 2 which reaches surface 1.

Other names for the radiation shape factor are *view factor, angle factor,* and *configuration factor.* The energy leaving surface 1 and arriving at surface 2 is

$$E_{b1}A_1F_{12}$$

and the energy leaving surface 2 and arriving at surface 1 is

$$E_{b2}A_2F_{21}$$

Since the surfaces are black, all the incident radiation will be absorbed, and the net energy exchange is

$$E_{b1}A_1F_{12} - E_{b2}A_2F_{21} = Q_{1-2}$$

If both surfaces are at the same temperature, there can be no heat exchange, that is, $Q_{1-2} = 0$. Also, for $T_1 = T_2$

$$E_{b1} = E_{b2}$$

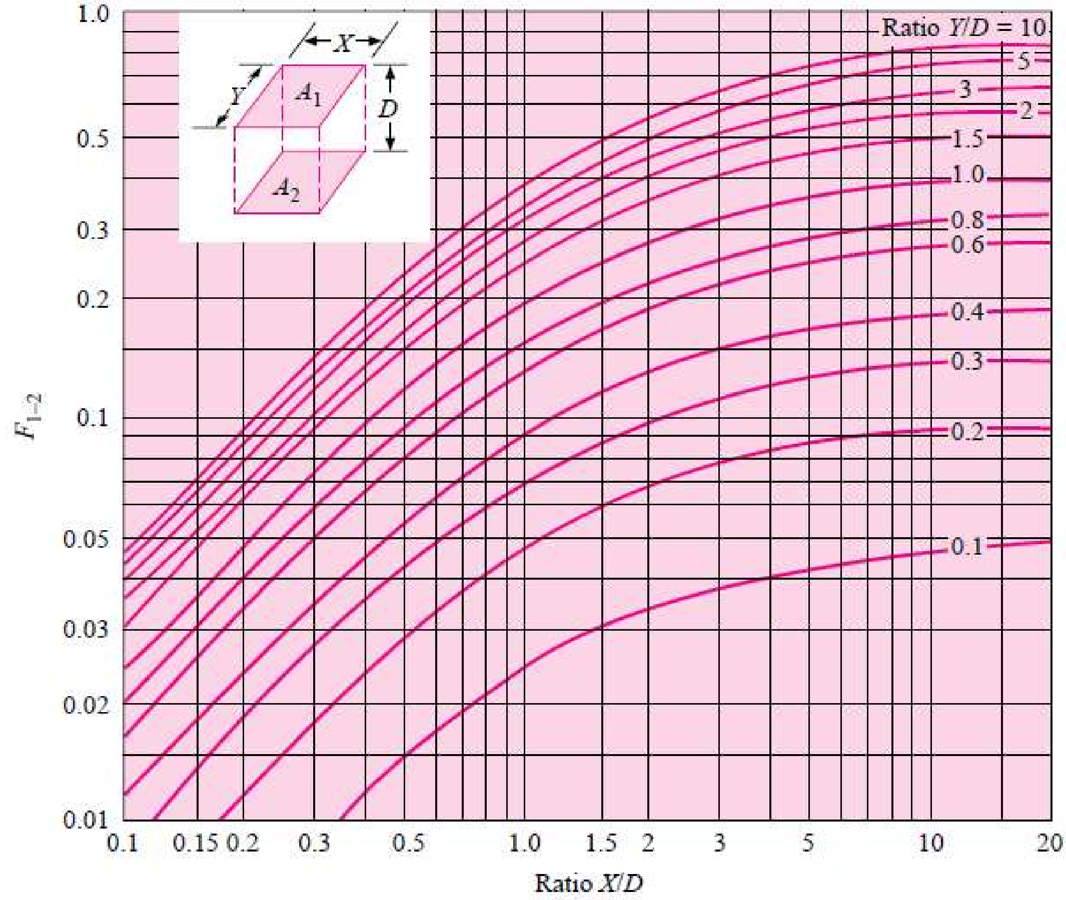
so that

$$A_1F_{12} = A_2F_{21} \qquad \textbf{[8-18]}$$

The net heat exchange is therefore

$$Q_{1-2} = A_1F_{12}(E_{b1} - E_{b2}) = A_2F_{21}(E_{b1} - E_{b2}) \qquad \textbf{[8-19]}$$

Figure 8-12 | Radiation shape factor for radiation between parallel rectangles.



Heat Transfer Between Black Surfaces

EXAMPLE 8-2

Two parallel black plates 0.5 by 1.0 m are spaced 0.5 m apart. One plate is maintained at 1000°C and the other at 500°C. What is the net radiant heat exchange between the two plates?

Solution

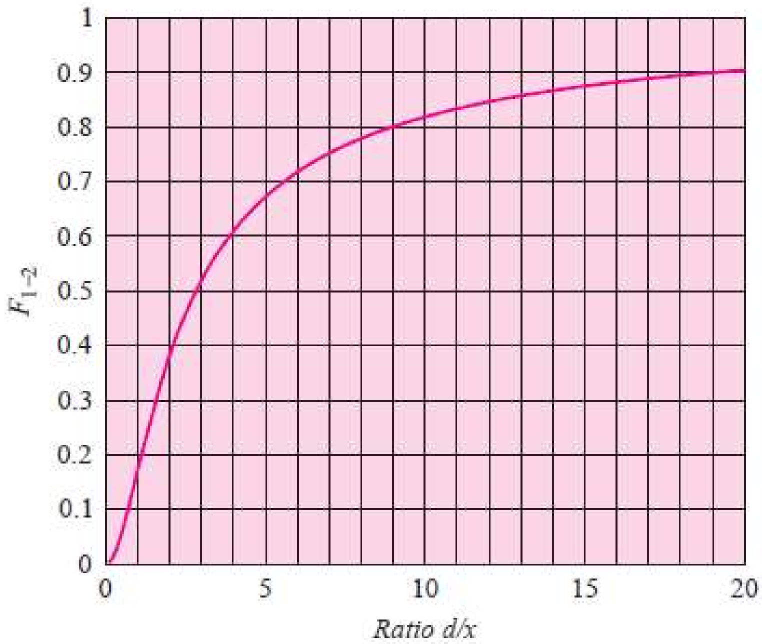
The ratios for use with Figure 8-12 are

$$\frac{Y}{D} = \frac{0.5}{0.5} = 1.0 \qquad \frac{X}{D} = \frac{1.0}{0.5} = 2.0$$

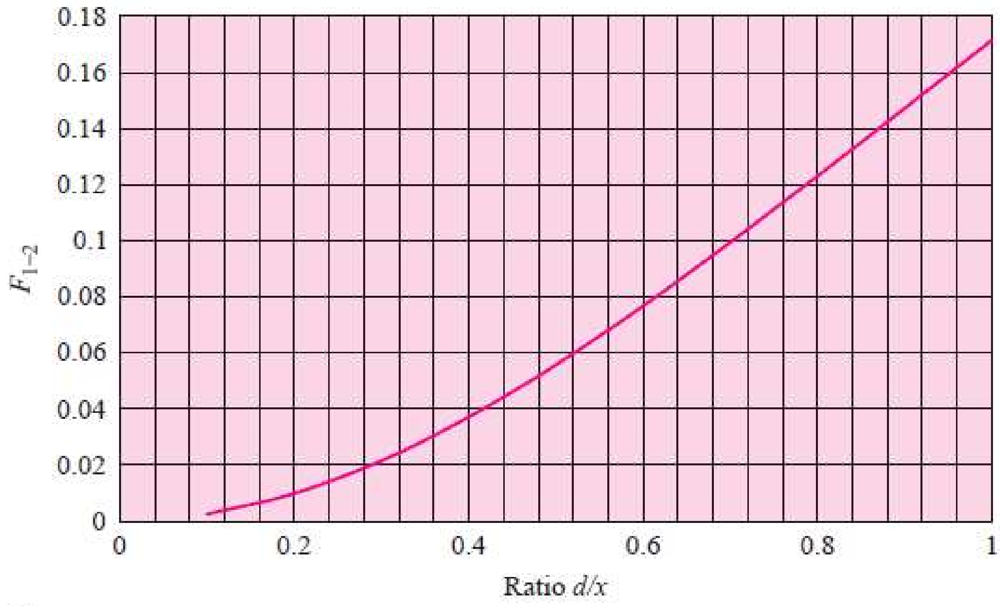
so that $F_{12} = 0.285$. The heat transfer is calculated from

$$\begin{aligned} q &= A_1 F_{12}(E_{b1} - E_{b2}) = \sigma A_1 F_{12}(T_1^4 - T_2^4) \\ &= (5.669 \times 10^{-8})(0.5)(0.285)(1273^4 - 773^4) \\ &= 18.33 \text{ kW} \quad [62{,}540 \text{ Btu/h}] \end{aligned}$$

Figure 8-13 | Radiation shape factor for radiation between parallel equal coaxial disks.



(a)



(b)

Figure 8-14 | Radiation shape factor for radiation between perpendicular rectangles with a common edge.

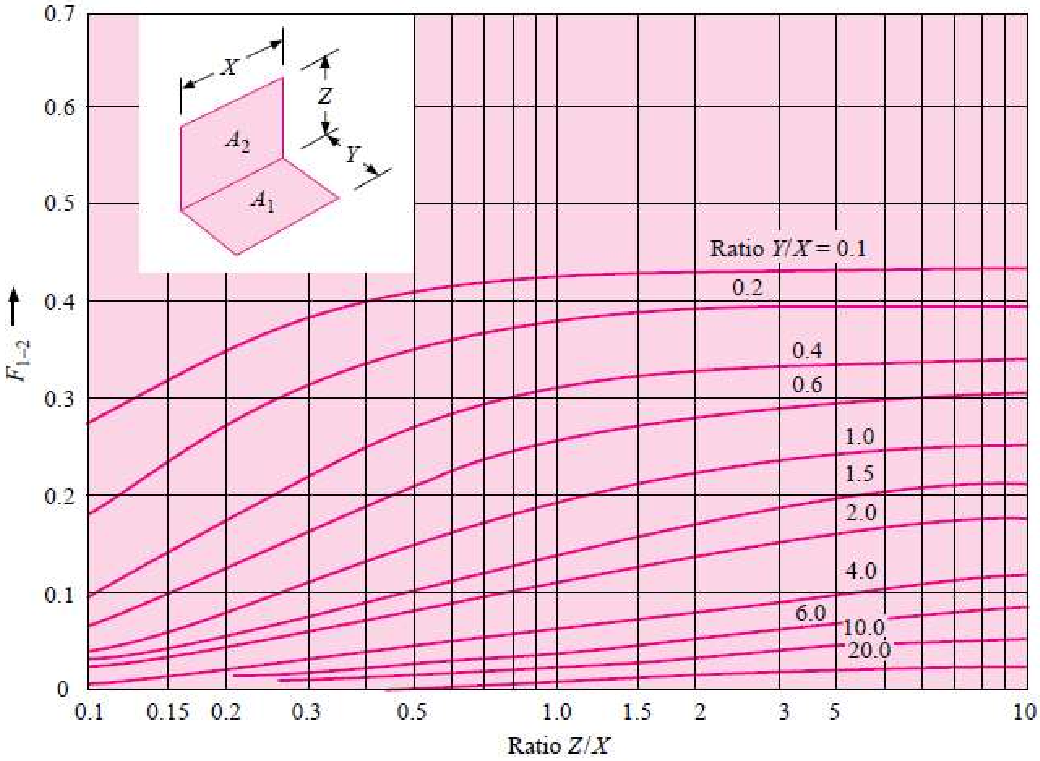
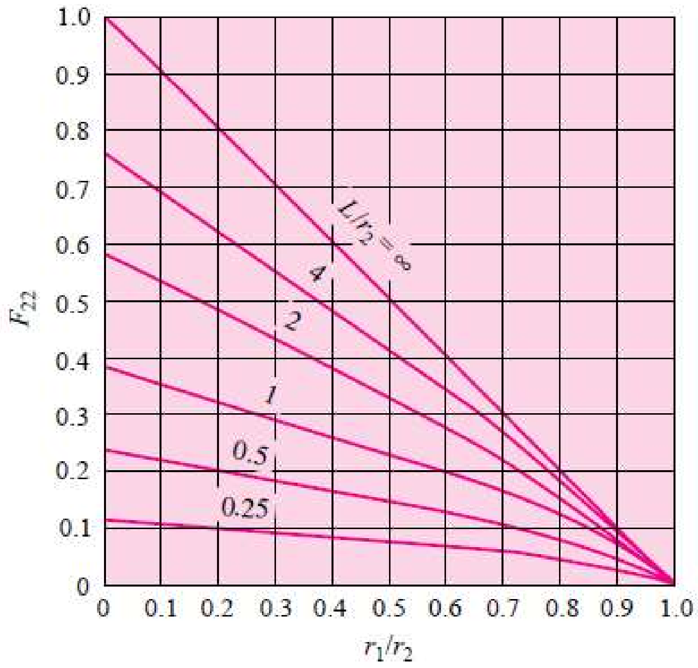
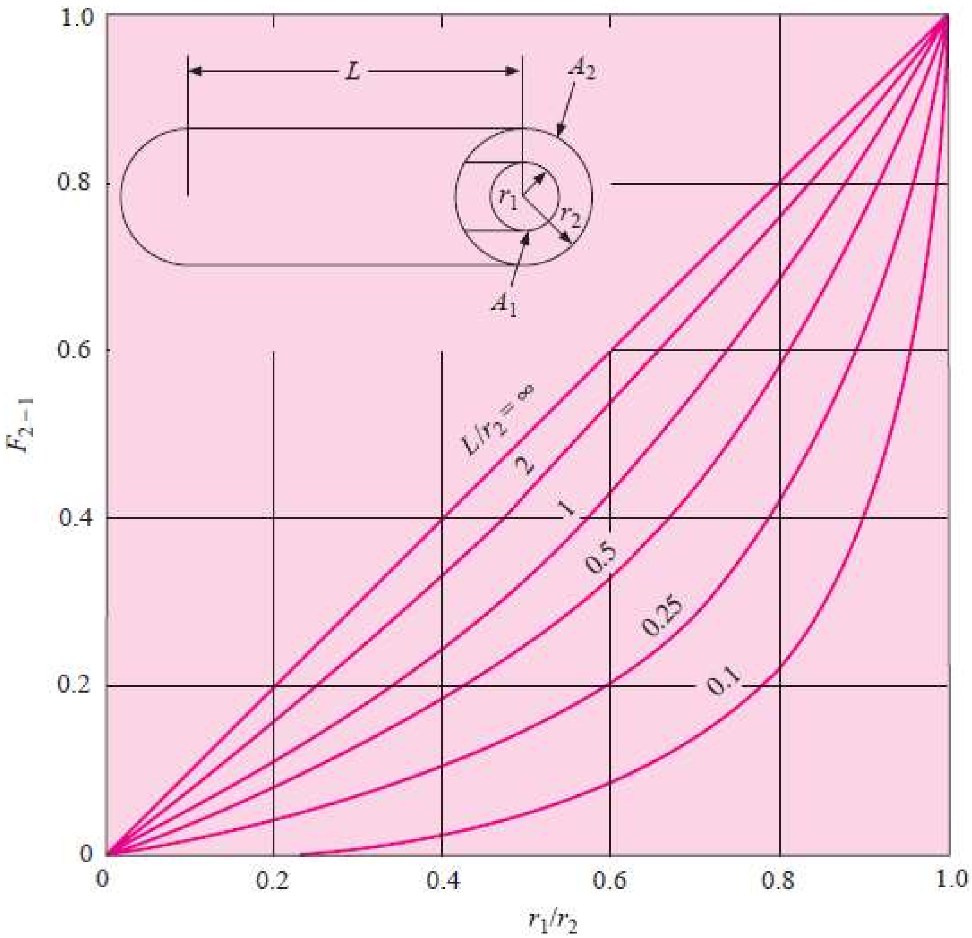


Figure 8-15 | Radiation shape factors for two concentric cylinders of finite length. (*a*) Outer cylinder to itself; (*b*) outer cylinder to inner cylinder.



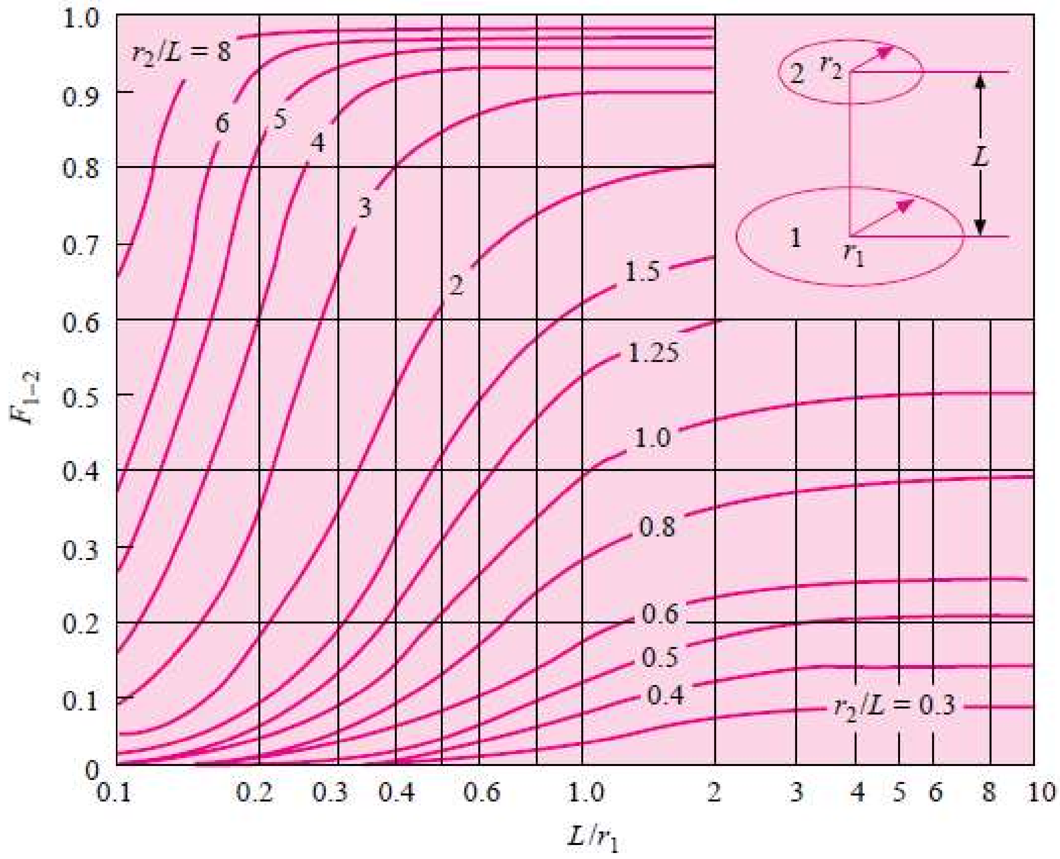
(*a*)



(*b*)

,

Figure 8-16 | Radiation shape factor for radiation between two parallel coaxial disks.



8-5 | RELATIONS BETWEEN SHAPE FACTORS

Some useful relations between shape factors may be obtained by considering the system shown in Figure 8-19. Suppose that the shape factor for radiation from A_3 to the combined area $A_{1,2}$ is desired. This shape factor must be given very simply as

$$F_{3-1,2} = F_{3-1} + F_{3-2} \quad [8\text{-}25]$$

that is, the total shape factor is the sum of its parts. We could also write Equation (8-25) as

$$A_3 F_{3-1,2} = A_3 F_{3-1} + A_3 F_{3-2} \quad [8\text{-}26]$$

and making use of the reciprocity relations

$$A_3 F_{3-1,2} = A_{1,2} F_{1,2-3}$$
$$A_3 F_{3-1} = A_1 F_{1-3}$$
$$A_3 F_{3-2} = A_2 F_{2-3}$$

the expression could be rewritten

$$A_{1,2} F_{1,2-3} = A_1 F_{1-3} + A_2 F_{2-3} \quad [8\text{-}27]$$

which simply states that the total radiation arriving at surface 3 is the sum of the radiations from surfaces 1 and 2. Suppose we wish to determine the shape factor F_{1-3} for the surfaces in Figure 8-20 in terms of known shape factors for perpendicular rectangles with a common edge. We may write

$$F_{1-2,3} = F_{1-2} + F_{1-3}$$

in accordance with Equation (8-25). Both $F_{1-2,3}$ and F_{1-2} may be determined from Figure 8-14, so that F_{1-3} is easily calculated when the dimensions are known. Now consider the somewhat more complicated situation shown in Figure 8-21. An expression for the shape factor F_{1-4} is desired in terms of known shape factors for perpendicular rectangles

Figure 8-19 | Sketch showing some relations between shape factors.

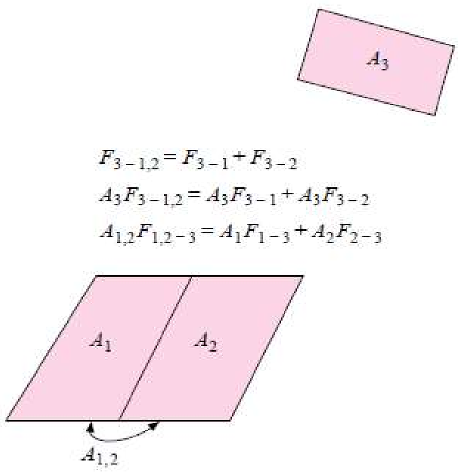


Figure 8-20

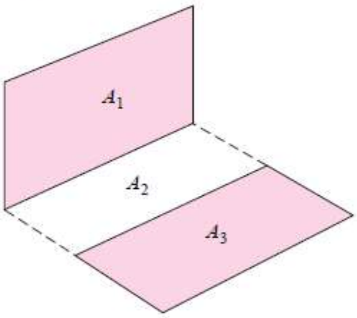
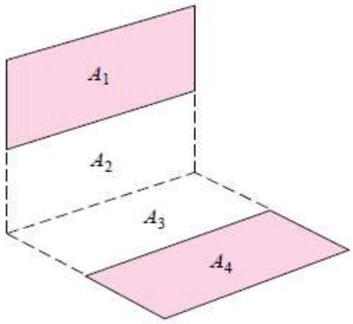


Figure 8-21



with a common edge. We write

$$A_{1,2}F_{1,2-3,4} = A_1F_{1-3,4} + A_2F_{2-3,4} \qquad [a]$$

in accordance with Equation (8-25). Both $F_{1,2-3,4}$ and $F_{2-3,4}$ can be obtained from Figure 8-14, and $F_{1-3,4}$ may be expressed

$$A_1F_{1-3,4} = A_1F_{1-3} + A_1F_{1-4} \qquad [b]$$

Also

$$A_{1,2}F_{1,2-3} = A_1F_{1-3} + A_2F_{2-3} \qquad [c]$$

Solving for A_1F_{1-3} from (c), inserting this in (b), and then inserting the resultant expression for $A_1F_{1-3,4}$ in (a) gives

$$A_{1,2}F_{1,2-3,4} = A_{1,2}F_{1,2-3} - A_2F_{2-3} + A_1F_{1-4} + A_2F_{2-3,4} \qquad [d]$$

Notice that all shape factors except F_{1-4} may be determined from Figure 8-14. Thus

$$F_{1-4} = \frac{1}{A_1}(A_{1,2}F_{1,2-3,4} + A_2F_{2-3} - A_{1,2}F_{1,2-3} - A_2F_{2-3,4}) \qquad [8\text{-}28]$$

In the foregoing discussion the tacit assumption has been made that the various bodies do not see themselves, that is,

$$F_{11} = F_{22} = F_{33} = 0 \cdots$$

To be perfectly general, we must include the possibility of concave curved surfaces, which may then see themselves. The general relation is therefore

$$\sum_{j=1}^{n} F_{ij} = 1.0 \qquad [8\text{-}29]$$

where F_{ij} is the fraction of the total energy leaving surface i that arrives at surface j. Thus for a three-surface enclosure we would write

$$F_{11} + F_{12} + F_{13} = 1.0$$

and F_{11} represents the fraction of energy leaving surface 1 that strikes surface 1. A certain amount of care is required in analyzing radiation exchange between curved surfaces.

Shape-Factor Algebra for Open Ends of Cylinders

EXAMPLE 8-3

Two concentric cylinders having diameters of 10 and 20 cm have a length of 20 cm. Calculate the shape factor between the open ends of the cylinders.

Solution

We use the nomenclature of Figure 8-15 for this problem and designate the open ends as surfaces 3 and 4. We have $L/r_2 = 20/10 = 2.0$ and $r_1/r_2 = 0.5$; so from Figure 8-15 or Table 8-2 we obtain

$$F_{21} = 0.4126 \qquad F_{22} = 0.3286$$

Using the reciprocity relation [Equation (8-18)] we have

$$A_1 F_{12} = A_2 F_{21} \qquad \text{and} \qquad F_{12} = (d_2/d_1)F_{21} = (20/10)(0.4126) = 0.8253$$

For surface 2 we have

$$F_{21} + F_{22} + F_{23} + F_{24} = 1.0$$

From symmetry $F_{23} = F_{24}$ so that

$$F_{23} = F_{24} = \left(\frac{1}{2}\right)(1 - 0.4126 - 0.3286) = 0.1294$$

Using reciprocity again,

$$A_2 F_{23} = A_3 F_{32}$$

and

$$F_{32} = \frac{\pi(20)(20)}{\pi(20^2 - 10^2)/4} 0.1294 = 0.6901$$

We observe that $F_{11} = F_{33} = F_{44} = 0$ and for surface 3

$$F_{31} + F_{32} + F_{34} = 1.0 \qquad [a]$$

So, if F_{31} can be determined, we can calculate the desired quantity F_{34}. For surface 1

$$F_{12} + F_{13} + F_{14} = 1.0$$

and from symmetry $F_{13} = F_{14}$ so that

$$F_{13} = \left(\frac{1}{2}\right)(1 - 0.8253) = 0.0874$$

Using reciprocity gives

$$A_1 F_{13} = A_3 F_{31}$$

$$F_{31} = \frac{\pi(10)(20)}{\pi(20^2 - 10^2)/4} 0.0874 = 0.233$$

Then, from Equation (a)

$$F_{34} = 1 - 0.233 - 0.6901 = 0.0769$$

EXAMPLE 8-4 Shape-Factor Algebra for Truncated Cone

A truncated cone has top and bottom diameters of 10 and 20 cm and a height of 10 cm. Calculate the shape factor between the top surface and the side and also the shape factor between the side and itself.

Solution

We employ Figure 8-16 for solution of this problem and take the nomenclature as shown, designating the top as surface 2, the bottom as surface 1, and the side as surface 3. Thus, the desired quantities are F_{23} and F_{33}. We have $L/r_1 = 10/10 = 1.0$ and $r_2/L = 5/10 = 0.5$. Thus, from Figure 8-16

$$F_{12} = 0.12$$

From reciprocity [Equation (8-18)]

$$A_1 F_{12} = A_2 F_{21}$$

$$F_{21} = (20/10)^2 (0.12) = 0.48$$

and

$$F_{22} = 0$$

so that

$$F_{21} + F_{23} = 1.0$$

and

$$F_{23} = 1 - 0.48 = 0.52$$

For surface 3,

$$F_{31} + F_{32} + F_{33} = 1.0 \qquad [a]$$

so we must find F_{31} and F_{32} in order to evaluate F_{33}. Since $F_{11} = 0$, we have

$$F_{12} + F_{13} = 1.0 \quad \text{and} \quad F_{13} = 1 - 0.12 = 0.88$$

and from reciprocity

$$A_1 F_{13} = A_3 F_{31} \qquad [b]$$

The surface area of the side is

$$A_3 = \pi(r_1 + r_2)\left[(r_1 - r_2)^2 + L^2\right]^{1/2}$$

$$= \pi(5 + 10)(5^2 + 10^2)^{1/2} = 526.9 \text{ cm}^2$$

So, from Equation (*b*)

$$F_{31} = \frac{\pi(10^2)}{526.9} 0.88 = 0.525$$

A similar procedure applies with surface 2 so that

$$F_{32} = \frac{\pi(5)^2}{526.9} 0.52 = 0.0775$$

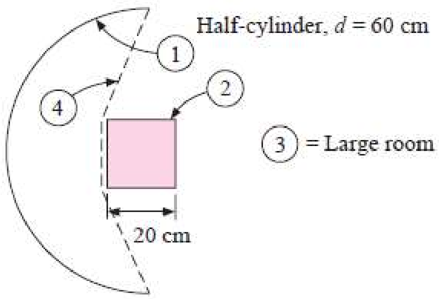
Finally, from Equation (*a*)

$$F_{33} = 1 - 0.525 - 0.0775 = 0.397$$

Shape-Factor Algebra for Cylindrical Reflector — EXAMPLE 8-5

The long circular half-cylinder shown in Figure Example 8-5 has a diameter of 60 cm and a square rod 20 by 20 cm placed along the geometric centerline. Both are surrounded by a large enclosure. Find F_{12}, F_{13}, and F_{11} in accordance with the nomenclature in the figure.

Figure Example 8-5



■ **Solution**

From symmetry we have

$$F_{21} = F_{23} = 0.5 \qquad [a]$$

In general, $F_{11} + F_{12} + F_{13} = 1.0$. To aid in the analysis we create the fictitious surface 4 shown as the dashed line. For this surface, $F_{41} = 1.0$. Now, all radiation leaving surface 1 will arrive either at 2 or at 3. Likewise, this radiation will arrive at the imaginary surface 4, so that

$$F_{14} = F_{12} + F_{13} \qquad [b]$$

From reciprocity,

$$A_1 F_{14} = A_4 F_{41}$$

The areas are, for unit length,

$$A_1 = \pi d/2 = \pi(0.6)/2 = 0.942$$
$$A_4 = 0.2 + (2)[(0.1)^2 + (0.2)^2]^{1/2} = 0.647$$
$$A_2 = (4)(0.2) = 0.8$$

so that

$$F_{14} = \frac{A_4}{A_1} F_{41} = \frac{(0.647)(1.0)}{0.942} = 0.686 \qquad [c]$$

We also have, from reciprocity,

$$A_2 F_{21} = A_1 F_{12}$$

so

$$F_{12} = \frac{A_2}{A_1} F_{21} = \frac{(0.8)(0.5)}{0.942} = 0.425 \qquad [d]$$

Combining (*b*), (*c*), and (*d*) gives

$$F_{13} = 0.686 - 0.425 = 0.261$$

Finally,

$$F_{11} = 1 - F_{12} - F_{13} = 1 - 0.425 - 0.261 = 0.314$$

This example illustrates how one may make use of clever geometric considerations to calculate the radiation shape factors.

8-6 | HEAT EXCHANGE BETWEEN NONBLACKBODIES

In addition to the assumptions stated above, we shall also assume that the radiosity and irradiation are uniform over each surface. This assumption is not strictly correct, even for ideal gray diffuse surfaces, but the problems become exceedingly complex when this analytical restriction is not imposed. Sparrow and Cess [10] give a discussion of such problems. As shown in Figure 8-24, the radiosity is the sum of the energy emitted and the energy reflected when no energy is transmitted, or

$$J = \epsilon E_b + \rho G \qquad \textbf{[8-36]}$$

where ϵ is the emissivity and E_b is the blackbody emissive power. Since the transmissivity is assumed to be zero, the reflectivity may be expressed as

$$\rho = 1 - \alpha = 1 - \epsilon$$

so that

$$J = \epsilon E_b + (1 - \epsilon)G \qquad \textbf{[8-37]}$$